

Virtual workshop on Geometric representation theory

---with a focus on flag varieties and affine Grassmanians

This workshop aims to bring together young experts working on the geometry of flag varieties and affine Grassmanians, its connections with various representation theories of groups and algebras of Lie type, and the geometric Langlands program. This workshop also aims to promote the development of geometric representation theory in China.

Zoom: <https://unc.zoom.us/j/91048352299>

Passcode: 314159

Time:

July 25-29, 8:00-10:30 am/pm (Beijing time)

Speakers :

Bao Huanchen (National University of Singapore)

Marc Besson (BICMR, Peking University)

Chen Tsao-Hsien (University of Minnesota)

Chen Zhe (Shantou University)

He Xuhua (The Chinese University of Hong Kong)

Li Penghui (YMSC, Tsinghua University)

Shan Peng (YMSC, Tsinghua University)

Shen Linhui (Michigan State University)

Shu Cheng (IASM, Zhejiang University)

Su Changjian (YMSC, Tsinghua University)

Xu Daxin (AMSS, CAS)

Xue Ting (University of Melbourne)

Yang Ruotao (Skoltech)

Yi Lingfei (University of Minnesota)

Yu Jize (The Chinese University of Hong Kong)

Yu Shilin (Xiamen University)

Yun Zhiwei (MIT)

Zhao Gufang (University of Melbourne)

Zhao Yifei (Universität Münster)

Zhou Yehao (Perimeter Institute)

Organizers :

Hong Jiuzu (UNC Chapel Hill)

Nie Sian (AMSS CAS)

Schedule

Beijing time	July 25	July 26	July 27	July 28	July 29
8-9am	Zhao Gufang	Yu Shilin	Zhou Yehao	Yu Jize	Shu Cheng
9:15-10:15am	Shan Peng	Li Penghui	Xu Daxin	Marc Besson	Yun Zhiwei
8-9pm	Su Changjian	Yang Ruotao	Chen Zhe	He Xuhua	Zhao Yifei
9:15-10:15pm	Xue Ting	Shen Linhui	Yi Lingfei	Chen Tsao-Hsien	Bao Huanchen

Titles/Abstracts:

July 25, Monday

Zhao Gufang (8-9am, Mon)

Towards higher loop Grassmannians via fusion

This is based on a work in progress, in collaboration with Ivan Mirkovic and Yaping Yang. we define a version of factorization spaces over Hilbert scheme of points on a smooth algebraic variety M , as a refinement of Beilinson-Drinfeld factorization spaces. When M is an algebraic surface, an example of factorization space will be given, as a quote scheme of torsion free sheaves on M . The factorization structure is constructed building up on the work of Haiman on Hilbert schemes, as well as the work of Feigin-Loktev and Chari-Pressley on local Weyl modules. Global sections of a tautological line bundle on this factorization space yield a local Weyl module of the toroidal algebra, whose characters are given by Macdonald polynomials.

Shan Peng (9:15-10:15am, Mon)

A geometric realization of the center of small quantum groups

We will explain a geometric realization of the center of small quantum groups using cohomology of affine Springer fibers. This is based on joint work with R. Bezrukavnikov, P. Boixeda-Alvarez, and Eric Vasserot.

Su Changjian (8-9pm, Mon)

Motivic Chern classes of Schubert cells and applications

Motivic Chern classes in K -theory are generalizations of MacPherson classes in homology. I will talk about some recent developments about motivic Chern classes of the Schubert cells in the flag varieties,

including their description using Hecke algebra, relation to the stable envelopes, applications in the representations of p -adic dual groups, and so on.

Xue Ting (9:15-10:15pm, Mon)

Character sheaves and Hessenberg varieties

The Springer theory relates nilpotent orbits in the Lie algebra of a reductive algebraic group to irreducible representations of Coxeter groups. We discuss a Springer theory for symmetric pairs and describe the character sheaves arising in this setting. Via a nearby cycle construction irreducible representations of Hecke algebras with parameters $1, -1$ enter the description. We explain its application to cohomology of Hessenberg varieties, examples of which include classical objects in algebraic geometry such as intersections of quadrics. If time permits, we discuss connections with the work of Lusztig and Yun relating character sheaves to irreducible representations of trigonometric double affine Hecke algebras. The talk is based on joint work with K. Vilonen and partly with T.H. Chen and M. Grinberg.

July 26, Tuesday

Yu Shilin (8-9am, Tue)

Coadjoint orbit method via deformation quantization

The coadjoint orbit method of Kirillov and Kostant suggests that irreducible unitary representations of a Lie group can be constructed as "quantizations" of coadjoint orbits of the group. In the (most difficult) case of noncompact semisimple Lie groups (still open), it is reformulated by Vogan in terms of quantization of equivariant vector bundles on nilpotent coadjoint orbits. In this talk, I propose a new scheme to quantize nilpotent orbits using deformation quantization of symplectic varieties and their Lagrangian subvarieties, which works uniformly for any semisimple group, classical or exceptional, linear or non-linear. This is partially based on a joint paper with Conan Leung and ongoing project with Ivan Losev.

Li Penghui (9:15-10:15am, Tue)

Revisiting mixed geometry

I will present joint work with Quoc P. Ho on our theory of graded sheaves which provides a uniform construction of "mixed versions" or "graded lifts" in the sense of Beilinson--Ginzburg--Soergel which works for arbitrary Artin stacks. In particular, we obtain a general construction of graded lifts of many categories arising in geometric representation theory and categorified knot invariants. Our sheaf theory comes with a six-functor formalism, a perverse t -structure in the sense of Beilinson--Bernstein--Deligne--Gabber, and a weight (or co- t -)structure in the sense of Bondarko and Pauksztello, all compatible, in a precise sense, with the six-functor formalism, perverse t -structures, and Frobenius weights on l -adic

sheaves. Historically, constructions of graded lifts were done on a case-by-case basis and were technically subtle, due to Frobenius' non-semisimplicity. Our construction sidesteps this issue by semi-simplifying the Frobenius action itself. However, our categories agree with those previously constructed when they are available. For example, the monoidal DG-category of chain complexes of Soergel bimodules is equivalent to the category of constructible graded sheaves on $B\backslash G/B$.

Yang Ruotao (8-9pm, Tue)

Untwisted Gaiotto equivalence for $GL(M|N)$

This is a joint work with Roman Travkin. A conjecture of Davide Gaiotto predicts that the category of representations of quantum supergroup $U_q(\mathfrak{gl}(M|N))$ can be realized as a category of twisted D-modules with certain equivariant condition on the affine Grassmannian Gr_N . The untwisted version of the above conjecture says that the category of representations of the degenerate supergroup is equivalent to the category of (non-twisted) D-modules, with the same equivariant condition on Gr_N . In the case of $M=N-1$ and $M=N$, the latter was proved by A. Braverman, M. Finkelberg, V. Ginzburg, and R. Travkin. In this talk, we prove all other cases.

If time permits, we will also introduce recent progress on the twisted Gaiotto equivalence.

Shen Linhui (9:15-10:15pm, Tue)

Cluster structures on braid varieties

Let G be a complex simple group of type ADE. Let β be a positive braid whose Demazure product is the longest Weyl group element. The braid variety $M(\beta)$ is a moduli space parametrizing chains of flags with relative position prescribed by β . The variety $M(\beta)$ generalizes many well known varieties, including positroid cells, open Richardson varieties, and double Bott-Samelson cells. We provide a concrete construction of the cluster structures on $M(\beta)$, using the weaves of Casals and Zaslow. We show that the coordinate ring of $M(\beta)$ is a cluster algebra, which confirms a conjecture of Leclerc as special cases. If time permits, I will explain several of its applications on representation theory and knot theory, including its connections with the Kazhdan-Lusztig R-polynomials and a geometric interpretation of the Khovanov-Rozansky homology (following the work of Lam-Speyer and Galashin-Lam). This talk is based on joint work with Roger Casals, Eugene Gorsky, Mikhail Gorsky, Ian Le, and Jose Simental.

July 27, Wednesday

Zhou Yehao (8-9am, Wed)

Stratification and transversal slices in the generalized affine Grassmannian slices

Generalized affine Grassmannian slices are introduced by Braverman, Finkelberg and Nakajima, they arise as Coulomb branches of 3d $N=4$ quiver gauge theories. In this talk I will review the definition of generalized affine Grassmannian slices, their stratifications, and then prove a description of the transversal slices to these strata, previously conjectured by BFN.

Xu Daxin (9:15-10:15am, Wed)

Drinfeld Lemma for F-isocrystals

Drinfeld's lemma for l -adic local systems is a fundamental result in arithmetic geometry. It plays an important role in the Langlands correspondence for a reductive group over the function field of a curve over a finite field, pioneered by Drinfeld for GL_2 and subsequently extended by L. Lafforgue and then V. Lafforgue. In this talk, we will discuss Drinfeld's lemma for p -adic local systems: overconvergent/convergent F-isocrystals. This is based on a joint work with Kiran Kedlaya.

Chen Zhe (8-9pm, Wed)

On orbits and generic higher Deligne--Lusztig representations

In this talk we would like to discuss an algebraisation problem in the representation theory of (connected) reductive groups over discrete valuation rings. This includes, in particular, a reduction theorem of odd level Deligne--Lusztig representations, which in turn implies that these geometrically constructed representations are all in regular semisimple orbits. This is based on a joint work in progress with Stasinski.

Yi Lingfei (9:15-10:15pm, Thu)

Physically rigidity of Frenkel-Gross connection

We show that the Frenkel-Gross connection is physically rigid, thus confirming the de Rham version of a conjecture of Heinloth-Ngô-Yun. The proof is based on the construction of the Hecke eigensheaf of a connection with only generic oper structure, using the localization of Weyl modules. In the talk, we will review the notion of opers and give the sketch of the proof.

July 28, Thursday

Yu Jize (8-9am, Thu)

The integral coefficient geometric Satake equivalence in mixed characteristic

The geometric Satake equivalence establishes a link between two symmetric monoidal categories of fundamental importance: the category of perverse sheaves on the local Hecke stack and the category of finitely generated representations of the Langlands dual group. It has many important applications in the study of the geometric Langlands program and number theory. In this talk, I will discuss the integral coefficient geometric Satake equivalence in the mixed characteristic setting which is established independent of Scholze's perfectoid theory. As a result of this equivalence, will present an arithmetic

application: a geometric construction of Jacquet-Langlands transfer for automorphic forms of higher weights.

Marc Besson (9:15-10:15am, Thu)

An introduction to the Frenkel-Kac isomorphism, affine Demazure modules and T-fixed subschemes

In this introductory talk I will describe the decomposition of the basic representation (in affine types ADE) into modules for the homogeneous Heisenberg algebra, originally due to Frenkel-Kac, as well as a geometric reinterpretation of this isomorphism due to Zhu. Zhu's isomorphism admits a refinement describing affine Demazure modules in terms of T-fixed subschemes; this result is due to Zhu in types A and D and completed by Besson and Hong in type E6. I will outline the proof and discuss the remaining open cases in E7 and E8. Time permitting, I will also describe an application of these results to the determination of the smooth locus of twisted affine Schubert varieties.

He Xuhua (8-9pm, Thu)

From a mysterious identity to the irreducible components of affine Deligne-Lusztig varieties

I will discuss the following mysterious identity

$$\sum_{\substack{k \geq 1, 1 > \frac{a_1}{b_1} > \dots > \frac{a_k}{b_k} > 0; \\ a_i + \dots + a_k = i, b_1 + \dots + b_k = n}} (q-1)^{(k-1)} q^{k-1 - \frac{\sum_{1 \leq l_1 < l_2 \leq k} (a_{l_1} b_{l_2} - a_{l_2} b_{l_1}) + \sum_{1 \leq l \leq k} \gcd(a_l, b_l)}{2}} = q^{\frac{i(n-i)-n}{2}}$$

and will explain how this identity helps us to understand the irreducible components of certain affine Deligne-Lusztig varieties in the affine flag varieties. This talk is based on the joint work with Sian Nie and Qingchao Yu.

Chen Tsao-Hsien (9:15-10:15pm, Wed)

Real groups, symmetric varieties, and derived Satake equivalence

In an ongoing project of D. Ben-Zvi, Y. Sakellaridis and A. Venkatesh, the authors propose a conjectural generalization of the derived geometric Satake equivalence for complex reductive groups to spherical varieties. I will describe a program aimed at establishing their conjecture in the case of symmetric varieties (an important class of spherical varieties). A key ingredient is the relation between the derived Satake equivalence for symmetric varieties and the geometric Langlands for real groups. This is joint work with David Nadler, Mark Macerato, John O'Brien.

July 29, Friday

Cheng Shu (8-9am, Fri)

Mixed Hodge polynomials of character varieties

Hausel, Letellier and Rodriguez-Villegas computed the E-polynomial of character varieties with generic semi-simple conjugacy classes. Their computation led to a conjectural formula for the mixed Hodge polynomial of character varieties. We will recall their results and introduce character varieties that are unitary in the global sense. The same method gives a conjectural formula for the mixed Hodge polynomial, which is built of Macdonald polynomials and wreath Macdonald polynomials.

Yun Zhiwei (9:15-10:15am, Fri)

Cocenter of the affine Hecke category and functions on the commuting stack

The Betti geometric Langlands conjecture proposed by Ben-Zvi and Nadler predicts that the cocenter of the affine Hecke category is equivalent to the category of automorphic sheaves on an elliptic curve. We explain how to prove the "semistable part" of this conjectural equivalence. As an application, we deduce a formula for the derived coordinate ring of the commuting stack for the Langlands dual group, proving a conjecture of Berest, Ramadoss and Yeung. This is joint work with Penghui Li and David Nadler.

Zhao Yifei (8-9pm, Fri)

Metaplectic spectral decomposition

The topological group $SL_2(\mathbb{R})$ has a unique double cover up to isomorphism. It is not a linear algebraic group and yet plays a vital role in arithmetics, giving representation-theoretic meaning to modular forms of half-integral weights. This topological group is an example of a "metaplectic group". In this talk, I will propose a general framework for such groups over an arbitrary base scheme, extending Deligne's work on the simply connected case. As an application, I will explain the Langlands parametrization of metaplectic cusp forms over a global function field.

Bao Huanchen (9:15-10:15pm, Fri)

Regularity theorem for totally non-negative flag varieties

The totally nonnegative flag variety was introduced by Lusztig. It has enriched combinatorial, geometric, and Lie-theoretic structures. In this talk, I will introduce a (new) J-total positivity on the full flag variety of an arbitrary Kac-Moody group, generalizing the (ordinary) total positivity. The J-totally nonnegative

flag variety enjoys many favourable properties like the ordinary totally nonnegative flag variety. Moreover, the J-total positivity on the full flag variety provides a model for the (ordinary) totally nonnegative partial flag variety. As a consequence, we prove that the closure of each (ordinary) totally positive Richardson variety is a regular CW complex homeomorphic to a closed ball, confirming conjectures of Galashin, Karp and Lam. This is based on joint work with Xuhua He.